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COMPUTER-SYNTHESIZED OPTICAL ELEMENTS IN DIAGNOSIS OF AEROSOL SYSTEMS

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Abstract—The problem of scattering of light by aerosol particles is investigated. The scattered field is represented as an expansion in two types of Bessel function corresponding to two different operations over the light fields. The coefficients of the expansion in these functions are shown to be directly related to the parameters characterizing the aerosol medium. A possibility is demonstrated for the practical evaluation of the aerosol characteristics with the aid of computer-synthesized optical elements. The scattered radiation pattern can be recovered from the particle parameter measurements.

Aerosol dispersive systems are suspensions of liquid or solid particles in gas. Interest in optical study of aerosols is motivated by their use in many fields, such as sprays and coatings, and important natural occurrences such as fogs, clouds and dust.

The optical methods based on solving the inverse scattering problem when the indicatrix of scattering is known are common in studies of dispersive media. However, the traditional optical methods are complicated both experimentally (the entire scattering indicatrix has to be measured) and theoretically (in the analysis of measurement data an ill-posed inverse problem has to be solved).

The present paper suggests an approach to the problem of scattering of light by aerosol particles which takes account of the symmetry of the problem. The key point of such an approach lies in the choice of a basis (a full set of functions) that satisfies the wave equation and over which the scattered field can be expanded. The series expansion in the basis selected in respect of the symmetry of the problem reduces the number of effectively involved basis functions and ensures fast convergence of the series. In addition, the coefficients of the expansion are directly related to the geometric parameters of the particles under study.

The characteristic feature of liquid aerosols is the axial symmetry of each particle. The basis solutions of the wave equation that describes scattering in this case are Bessel functions. Below we demonstrate that there exist two types of expansion in Bessel functions, which correspond to two operations over the light field. It is worth noting that these transformations have become practically feasible with the development of new computer-synthesized optical elements [1].

Consider the problem of scattering of radiation by an ensemble of spherical particles. In the Kirchhoff approximation, in the Fraunhofer zone the scattered field has the form [2]

$$E_s(\kappa) = \frac{1-m}{2\pi\kappa}\exp(-i\sqrt{k^2-\kappa^2 z})\sum_{i=1}^{N} A_0(r_i)a_i\exp(i\kappa r_i)J_1(\kappa a_i), \tag{1}$$

where $\kappa = k\sin\theta$, k is the wavenumber of the radiation, θ is the angle of scattering, m is the transmittance coefficient of a particle, $A_0(r_i)$ is the amplitude of the incident radiation, a_i is the radius of particle i, and $J_1(x)$ is the first-order Bessel function.

The Bessel functions are known to form a complete orthogonal system, that is (see, e.g. [3]),

$$\int \beta J_1(\rho_i\beta)J_1(\rho_F\beta)\,d\beta = \begin{cases} 0, & \text{for } \rho_i \neq \rho_F \\ \frac{1}{2}[J_1'(\rho_F)]^2, & \text{for } \rho_i = \rho_F \end{cases}. \tag{2}$$

Therefore, the scattered field E_s may be expanded in a series in such a full set of functions, *viz.*,

$$E_s = \sum_{i=1}^{\infty} f_1 J_1(\kappa a_i). \tag{3}$$

Comparison of Eqs (1) and (3) indicates that the coefficients f_i of the expansion are directly related to the dimensions of particles a_i. Hence, the evaluation of f_i is practically equivalent to the evaluation of the particle size distribution. The coefficients f_i may be directly measured by the computer-synthesized spatial filters [1] whose transmission is proportional to the superposition of the orthogonal functions $J_1(\kappa a_i)$.

Let us now look at the physical meaning of the expansion coefficients when the scattered field E_s are expanded in Bessel functions of order k, *viz.*,

$$E_s = \sum_k f_k J_k(x). \tag{4}$$

Making use of the "multiplication theorem" [3] for $J_1(\lambda t)$

$$J_1(\lambda t) = \sum_{k=0}^{\infty} \frac{1}{k!} J_{1+k}(t) \left(\frac{1-\lambda^2}{2} \right)^k t^k \tag{5}$$

we rewrite Eq. (1) as

$$E_s = \frac{1-m}{2\pi\tilde{\kappa}} \exp(-i\sqrt{k^2-\kappa^2 z}) \sum_{i=1}^{N} A_0(r_i) a_i \exp(i\kappa r_i) \cdot a_i \sum_{k=0} \frac{1}{k!} J_{1+k}(\tilde{\kappa}) \left(\frac{1-a_i^2}{2} \right)^k \tilde{\kappa}^k. \tag{6}$$

Averaging over the ensemble of particles yields

$$\langle E_s \rangle = \frac{1-m}{2\pi\tilde{\kappa}} \exp(-i\sqrt{k^2-\kappa^2 z}) \sum_{k=0}^{\infty} \frac{1}{k!} J_{1+k}(\tilde{\kappa}) \tilde{\kappa}^k \int a^2 F(a) \left(\frac{1-\tilde{a}^2}{2} \right)^k \mathrm{d}a, \tag{7}$$

where $F(a)$ is the particle size distribution function. The coefficients f_k are seen to be directly connected to the moments of the distribution function

$$M_k = a^k F(a) \, \mathrm{d}a$$

namely,

$$\langle E_s \rangle = C \left[M_2 J_1(\tilde{\kappa}) + \frac{M_2 - M_4}{2} J_2(\kappa) + \frac{M_2 - 2M_4 + M_6}{4} \tilde{\kappa}^2 J_3(\tilde{\kappa}) + \cdots \right], \tag{8}$$

where

$$C = \frac{1-m}{2\pi\tilde{\kappa}} \exp(-i\sqrt{k^2-\kappa^2} \cdot z), \qquad \tilde{\kappa} = \sin\theta, \qquad \tilde{a} = ka.$$

This expression may be useful in solving the inverse scattering problem. The scattered field can be recovered by the known moments of the particle size distribution. An immediate implication is that the scattered radiation pattern can be controlled by varying the particle distribution function. We note that the expression (8) contains the even moments only. The coefficients f_k corresponding to odd moments are all absent in the small-angle approximation. The odd moments appear if we include the terms of the next order in the expression of the total field scattered by a sphere (see, e.g. Vaganov and Katsenelenbaum [4]). However, the proportion of such particles is small in the large particle case under consideration.

Thus, the coefficients of the expansion of a scattered field in a series in Fourier–Bessel functions represent the relative concentration of particles or the particle size distribution. The coefficients of the expansion in a Bessel function series of different order represent the moments of the distribution function. These transformations are performed by spatial filters that allow direct evaluation of the particle parameters.

The results of this study may be used in the analysis of scattering of light by stellar particles, meteors, sea spray particles and crystals of ice in clouds. This approach may be found useful in some problems of radio wave scattering by ionospheric inhomogeneities or by irregular planetary terrain, and in problems of statistical antenna theory concerned with the diffraction by apertures with random boundaries or with antennae excited by random currents.

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